

Simulating Ballast Shear Strength from Large-Scale Triaxial Tests

Discrete Element Method

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The railroad ballast layer consists of discrete aggregate particles, and the discrete element method (DEM) is the most widely adopted numerical method to simulate the particulate nature of ballast materials and their particle interactions. Large-scale triaxial tests performed in the laboratory under controlled monotonic and repeated loading conditions are commonly considered the best means to measure macroscopic mechanical properties of ballast materials, such as strength, modulus, and deformation characteristics, directly related to load-carrying and drainage functions of the ballast layer in the field. A DEM modeling approach is described for railroad ballast with realistic particle shapes developed from image analysis to simulate large-scale triaxial compression tests on a limestone ballast material. The ballast DEM model captures the strength behavior from both the traditional slow and the rapid shear loading rate types of monotonic triaxial compression tests. The results of the experimental study indicated that the shearing rate had insignificant influence on the results of the triaxial compression tests. The results also showed that the incremental displacement approach captured the measured shearing response, yet could save significant computational resources and time. This study shows that the DEM simulation approach combined with image analysis has the potential to be a quantitative tool to predict ballast performance.

Ballast is an essential layer of the railroad track structure and primarily provides load distribution and drainage. Although ballast aggregate materials are commonly specified as uniformly graded in size with angular particle shapes and crushed faces, ballast engineering properties, such as aggregate type and gradation, particle shape, texture and angularity, and particle hardness and abrasion resistance, can often vary within certain specifications to influence overall track behavior and performance. Large-scale triaxial tests are traditionally performed in the laboratory to evaluate field monotonic and repeated loading effects on ballast behavior (1–5). Previous research on numerical modeling of ballast behavior in railroad tracks has focused on conducting simulations of aggregate particle assemblies using the discrete element method (DEM) to address the particulate nature of ballast gradations. These studies used spherical or clusters of spherical

particles for modeling convenience and also to keep the computational resources manageable (4, 5). However, the modeled particles do not reflect the actual ballast shapes for realistic microscopic interactions and the corresponding macroscopic behavior. Therefore, Tutumluer et al. introduced an image analysis–based three-dimensional (3-D) aggregate shape re-creation approach to represent individual ballast particle sizes and shapes and to model polyhedral particles for use in 3-D DEM simulations (6). A DEM simulation of large-scale triaxial tests with such polyhedral particle shapes is quite challenging because of the significant increase in computational cost required.

Ghaboussi and Barbosa (7) developed the first polyhedral 3-D DEM code, BLOCKS3D, for particle flow, and Nezami et al. (8) developed the second-generation polyhedral DEM code, BLOKS3D, from the work of Ghaboussi and Barbosa (7) at the University of Illinois at Urbana-Champaign. BLOKS3D includes vastly enhanced particle shape properties and contact detection methods that provide increased code speed to make DEM simulations affordable in a reasonable run time. Therefore, a realistic DEM simulation is accessible with the BLOKS3D code using the polyhedral elements generated from the image analysis results of ballast materials. Tutumluer et al. (6) used the University of Illinois aggregate image analyzer (UIAIA) to develop key particle morphological indices such as the flat and elongated ratio, the angularity index, and the surface texture index for the particle shapes.

The DEM approach was first calibrated by laboratory large-scale direct shear test results for ballast strength simulations (9). The calibrated DEM model was then used to model the strength and settlement behavior of railroad ballast for the effects of multiscale aggregate morphological properties (6, 10). More recent applications of the calibrated DEM model investigated ballast gradation (11) and fouling issues (9, 12), which are known to influence track performance. A successful field validation study was conducted with the ballast DEM simulation approach through constructing and monitoring field settlement records of four different ballast test sections and then comparing the measured ballast settlements under monitored train loadings with DEM model predictions (13). More recently, Lee et al. represented realistic drained and undrained responses of sands by using polyhedral DEM simulations of triaxial compression tests with BLOKS3D (14). They introduced new elements to represent the triaxial cell membrane and new procedures to simulate triaxial tests.

The development and initial results are presented of a ballast DEM simulation approach for modeling ballast shear strength behavior from large-scale triaxial compression tests. Both traditional slow and rapid shear loading approaches are adopted in the laboratory tests to investigate the effect of loading rate on ballast strength. For the DEM

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simulations, both continuous shearing and incremental displacement approaches are investigated for effectiveness to simulate the ballast specimen load response in the displacement-controlled mode during the monotonic loading triaxial strength tests (14).

OBJECTIVE AND SCOPE

The overall objective of the ongoing research at the University of Illinois has been to fully develop the DEM modeling capability as a quantitative track performance simulation tool to better understand (a) the nature of ballast particulate interactions through strength, modulus, and deformation testing; (b) complex ballast behavior under dynamic train loading regimes in the field; and (c) fouling and degradation trends and their impact on track performance. The research scope in this study primarily focused on testing and modeling of ballast shear strength behavior by using large-scale triaxial tests, which are traditionally performed in the laboratory to evaluate the effects of field monotonic loading controlled by shear strength as well as repeated loading effects, that is, modulus and permanent deformation behavior. A realistic railroad ballast model based on the DEM and aggregate image analysis was successfully adopted to simulate large-scale triaxial compression tests through the innovative use of membrane elements. Simulating the laboratory large-scale triaxial tests can help researchers to understand ballast particle interactions better and is a key step toward developing a realistic DEM model of field ballast performance.

EXPERIMENTAL STUDY

Large-Scale Triaxial Compression Test Device

A large-scale triaxial compression test device has recently been developed at the University of Illinois specifically for testing ballast size aggregate materials (Figure 1). The test sample dimensions are 30.5 cm (12 in.) in diameter and 61.0 cm (24 in.) in height. The acrylic test chamber is made of high-strength glass fiber with dimensions of 61.0 cm (24 in.) in diameter and 122.0 cm (48 in.) in height. An internal load cell (Honeywell Model 3174) with a capacity of 89 kN (20 kips) is placed on top of the specimen top platen. Three vertical linear variable differential transformers (LVDTs) are placed around the cylindrical test sample at 120-degree angles to measure the vertical deformations of the specimen from the three different side locations. Another LVDT can also be mounted on a circumferential chain wrapped around the specimen at midheight to measure the radial deformation of the test sample.

The monotonic loading triaxial compression tests for ballast strength were conducted at three different confining pressures in the displacement control mode: 68.9 kPa (10 psi), 137.8 kPa (20 psi), and 206.7 kPa (30 psi). The ballast strength tests were conducted at two different loading rates: slow conventional and rapid traffic induced. The shearing rate for the slow conventional strength test was adopted as 1% strain per minute, corresponding to 0.1016 mm/s, which is a common triaxial test shearing rate in standard soil mechanics or geotechnical engineering practice. Garg and Thompson evaluated the strength properties of granular materials under transportation vehicle loading at rather rapid monotonic loading rates in which the ram moved up to a maximum displacement of 38 mm (1.5 in.) per second (15). In this study, with the intent to also investigate



FIGURE 1 Ballast sample in University of Illinois large-scale triaxial test device.

the influence of higher traffic-induced loading rates on the large-scale triaxial strength test results of ballast materials, laboratory tests were conducted at both the rapid shear rate of 5% strain per second as well as the slow shear rate of 1% strain per minute. With the 61.0-cm (24-in.) high ballast specimen, these loading rates correspond to vertical ram movements of 30.5 mm (1.2 in.) per second and 6.1 mm (0.24 in.) per minute, respectively. Because the large movements of the ram caused instant bulging and shearing of ballast samples, in order not to damage the circumferential chain and the LVDT mounted on it, the LVDT was not used during the ballast strength tests.

Ballast Material Properties

The ballast material used in the strength tests was a clean limestone with 100% crushed aggregates. Figure 2 shows the gradation properties of the ballast material; the properties adequately met the No. 24 gradation requirements of the American Railway Engineering and Maintenance-of-Way Association. Besides the grain size distribution, aggregate shape properties, especially the flat and elongated ratio, the angularity index, and the surface texture index, are key indices quantified by the recently enhanced UIAIA (6). One full bucket of the ballast material was scanned and analyzed with

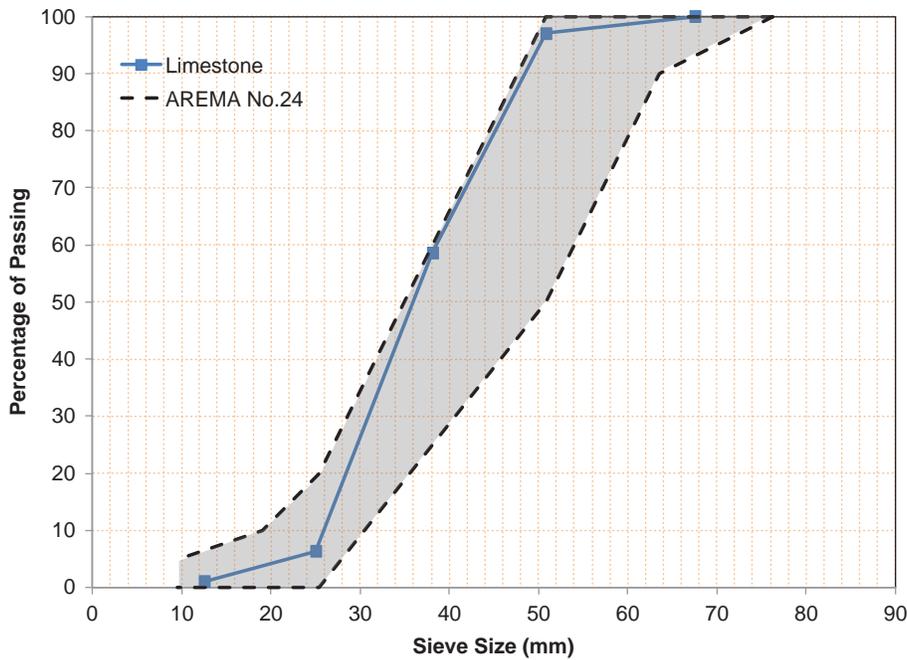


FIGURE 2 Gradation properties of limestone ballast material (AREMA = American Railway Engineering and Maintenance-of-Way Association).

the UIAIA to determine the values of the flat and elongated ratio, angularity index, and surface texture index. These shape indices were then used as the essential morphological data to generate ballast aggregate particle shapes as 3D discrete elements in the ballast DEM model (see Figure 3). The gradation properties and the average values of the limestone ballast shape indices used in the DEM simulations are as follows:

- Ballast type: limestone,
- Angularity index: 440 degrees,
- Flat and elongation ratio: 2.3,
- Surface texture: 2,
- Coefficient of uniformity (C_u): 1.46, and
- Coefficient of curvature (C_c): 0.97.

Ballast Sample Preparation

An aluminum split mold was used to prepare the ballast test samples. Three layers of a latex membrane, with a total thickness of 2.3 mm, were fixed inside the split mold and held in place by applying vacuum to prepare each specimen in layers. A thin layer of geotextile was placed on top of the base plate to prevent clogging of the vacuum pump. Approximately 68 to 73 kg (150 to 160 lb) of ballast material was poured into the mold evenly in four lifts, each lift constructed approximately 15 cm high (6 in.). The ballast sample was compacted with an electric jackhammer for about 4 s during construction of each lift. After all four lifts were compacted, the test sample was checked for the total height and the leveling of the top plate. A uniform compaction effort was applied for building each

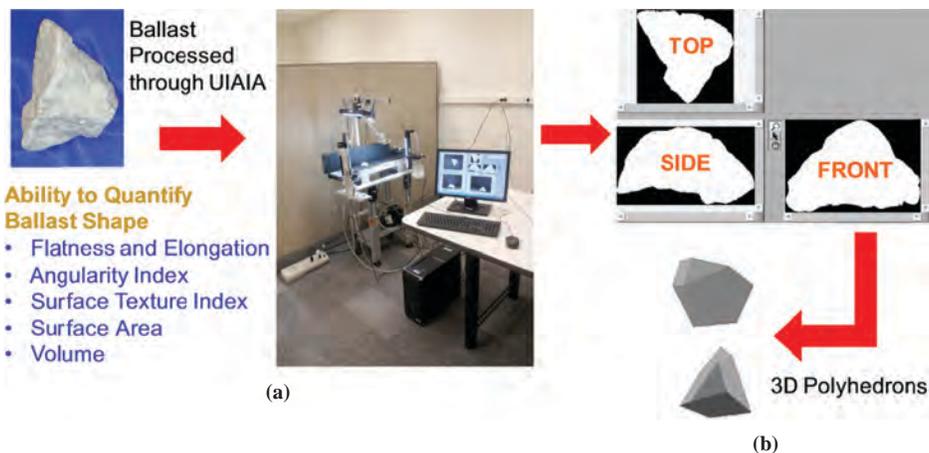


FIGURE 3 DEM for aggregate imaging-based railroad ballast: (a) enhanced UIAIA and (b) element built in DEM with desired shape properties.

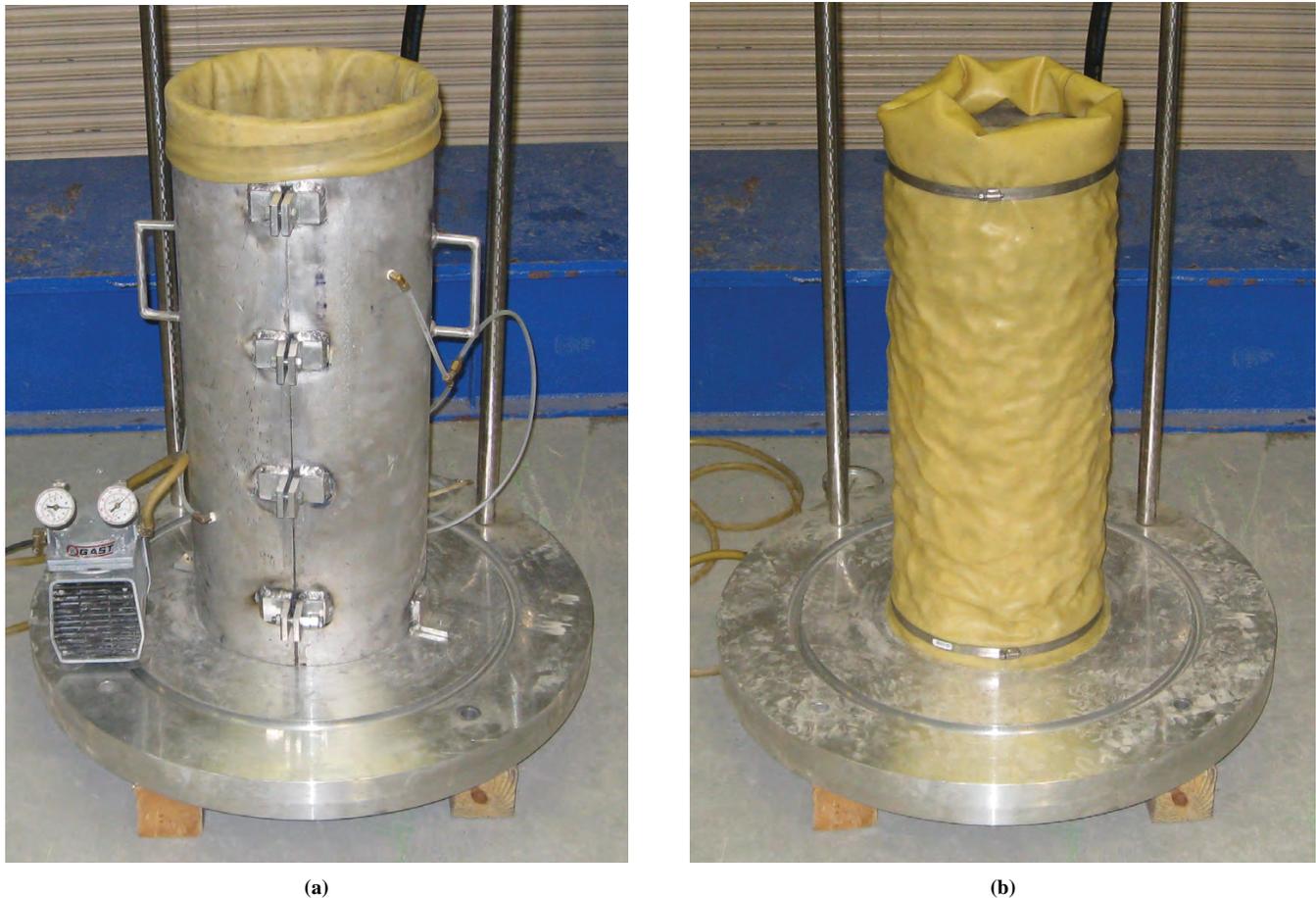


FIGURE 4 Large-scale triaxial test: (a) aluminum split mold and (b) ballast test sample.

limestone ballast specimen with an achieved target void ratio of around 0.68. Figure 4a shows the aluminum split mold and Figure 4b shows the compacted ballast sample for triaxial testing.

TEST RESULTS AND DISCUSSION

The results of the large-scale triaxial strength tests on the limestone ballast samples presented no significant differences for the different (slow and rapid) shearing rates adopted (Figure 5). As expected, the maximum deviator stress increased with increasing confining pressure and the test results show the initial slope of the stress–strain curve to be similar and consistent for the different confining pressures for both the slow and rapid shearing rates (Figure 5). In other words, the rapid shearing rate of 5% strain per second, corresponding to 30.5 mm/s (1.2 in./s), and the slow shearing rate of 1% strain per minute, corresponding to 0.102 mm/s, yielded similar stress–strain curves at target confining pressures of 68.9 and 137.8 kPa (10 and 20 psi). The slow shearing test for the 206.7-kPa (30-psi) target confining pressure was not performed and is not shown in Figure 5. Nevertheless, Figure 5 clearly indicates that the shearing rate is not a significant factor in these monotonic loading tests to influence the ballast strength results. The rapid shear test was previously introduced to better simulate traffic loading conditions in the field to determine the strength properties (15); however, such a test has less tolerance for specimen misalignment. The rapid test can also raise safety concerns in a laboratory

environment. However, a slow test is more advantageous in that the operator can better control the test boundary conditions.

DEM SIMULATIONS

Ballast DEM Model for Triaxial Compression Test

In this study, polyhedral particles were employed to represent realistic shapes of ballast aggregate particles in the DEM simulations. A polyhedral DEM code, BLOKS3D (16), was used to simulate the rigid polyhedral particles. The grain-size distribution and a set of particle shapes identified from the UIAIA analysis were used for user-defined inputs and the code then automatically generated polyhedral particles according to the gradation and shape library for the DEM simulation.

For the DEM simulations of a laboratory triaxial test, it is also necessary to model the flexible membrane (i.e., specimen latex membranes) other than the ballast specimen to be confined inside. Previous research efforts considered chains of circular or spherical particles to simulate the membrane (17–20). Lee et al. developed a variation of this approach for polyhedral DEM simulation: rigid rectangular cuboid discrete elements were positioned in a cylindrical arrangement to simulate the flexible membrane (14). This modeling approach was adopted in the current study. A total of 96 rigid rectangular cuboid discrete elements (in eight layers) were used to form a cylindrical membrane chamber to confine the ballast specimen, as shown in Figure 6.

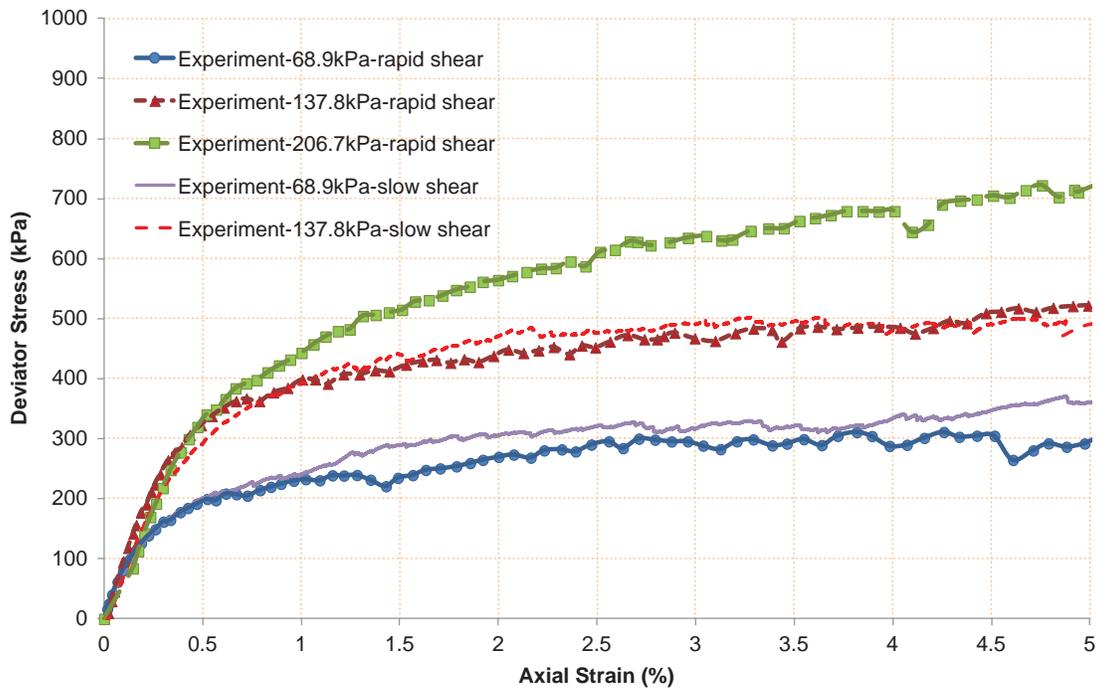


FIGURE 5 Laboratory ballast triaxial strength test results.

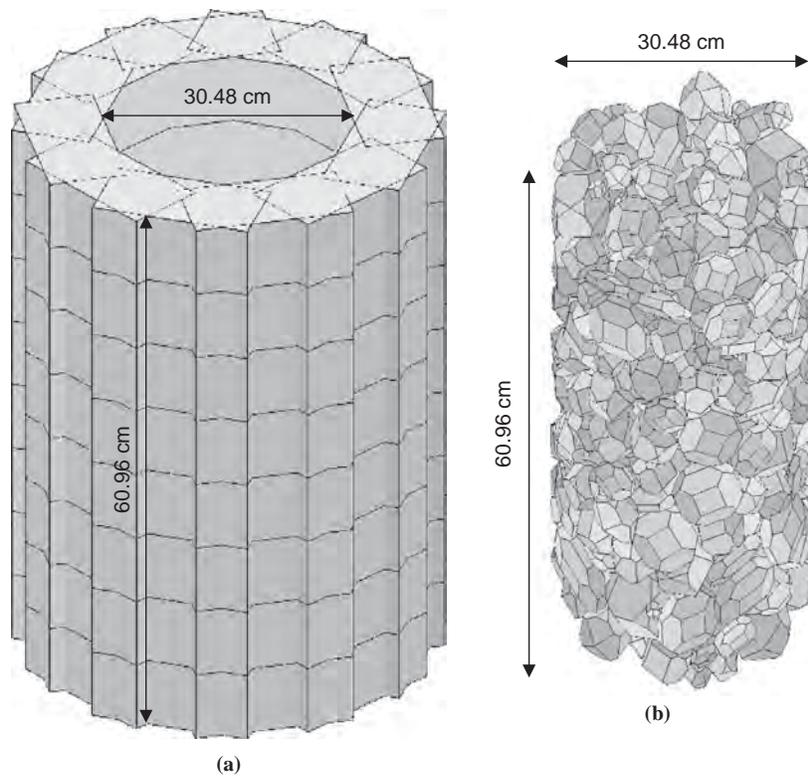


FIGURE 6 Triaxial test chamber used in DEM simulations: (a) flexible membrane to confine ballast specimen and (b) confined ballast sample.

Each layer had 12 equal-sized elements and each element was 20.32 cm (8 in.) long, 10.16 cm (4 in.) wide, and 7.62 cm (3 in.) high. These elements were only allowed to move in the radial direction. Rotations and translations in the other directions were restricted to replicate the deformation of the membrane. These elements were required to have certain thicknesses to avoid any gap between vertically adjacent layers when differential radial displacements between them were relatively large. Similarly, the elements were also allowed to have sufficient lengths to overlap horizontally adjacent elements and keep the circular chamber closed during simulation of the triaxial tests. No contact detection was made between these elements to allow for each element to move freely, independent of neighboring elements. The friction between the membrane elements and the ballast particles in contact with them was ignored during the DEM simulations.

After the membrane was formed, ballast particles were poured into the cylinder and the top platen was placed on top of the sample. In this way, around 500 particles were used to fill in the chamber. Compaction of the sample was then required to achieve the target density, as done in the laboratory experiment for each corresponding test sample under different confining pressures. Once the ballast DEM sample preparation was done, the shear loading could be performed by vertically moving a top platen. The 500 ballast particles were randomly generated in BLOKS3D, according to the gradation and shape library defined, so the initial configuration was different from one simulation to another. Therefore, the DEM simulations were repeated three times for each test scenario, and an average value of the three simulations was taken to efface the initial configuration effect. The details of the simulation parameters used in the DEM simulations are given in Table 1. The same DEM simulation parameters were used in this modeling study as the ones previously calibrated from the laboratory large-scale direct shear tests since the same limestone ballast material was tested in the previous study (9). Therefore, this test is a further proof that the DEM simulation approach developed at the University of Illinois is robust to simulate different experimental configurations of the same ballast material without recalibration and adjustment of the model parameters.

TABLE 1 Model Parameters Used in Ballast DEM Triaxial Test Simulations

DEM Model Parameter	Value
Interparticle friction angle	31°
Normal contact stiffness	20 MN/m
Shear contact stiffness	10 MN/m
Global damping	0.06
Contact damping	0.03
Time step	2.70×10^{-6} s
Ballast material density	2.65×10^3 kg/m ³

DEM Simulations with Continuous Shearing

Figure 7 presents the deformation of the ballast sample at different loading stages. The slow loading rate of 1% strain per minute was used in the DEM simulations with the continuous shearing method, which means that the top platen was vertically moving downward during the entire shearing stage at a constant speed of 0.102 mm/s until the target strain of 5%, corresponding to total displacement of 30.5 mm, was achieved. The DEM simulation was repeated three times for each confining pressure with the model parameters given in Table 1, and the maximum deviator stresses at failure or strength were recorded and the average value was taken for comparison with the experimental results. The continuous shearing method was not applied to model the response at higher rates. Further evaluation of DEM model parameters might be needed to calibrate them for rate independence as indicated from the experiments.

The DEM model predictions for stress–strain curves are compared in Figure 8 with the experimental results from the slow loading rate tests. Again, the slow shearing test for the 206.7-kPa (30-psi) target confining pressure is missing in Figure 8. Both the initial slopes of the stress–strain curves and the peak stresses from the DEM simulations show good agreement with the corresponding

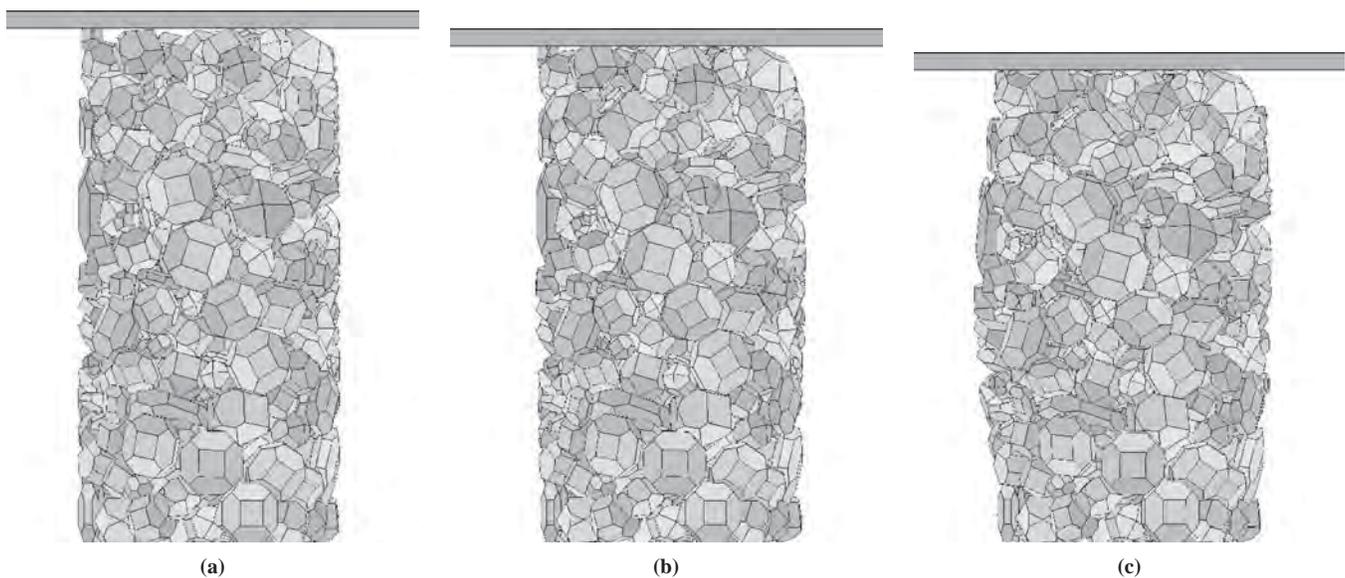


FIGURE 7 DEM simulations of ballast sample shown at different loading stages.

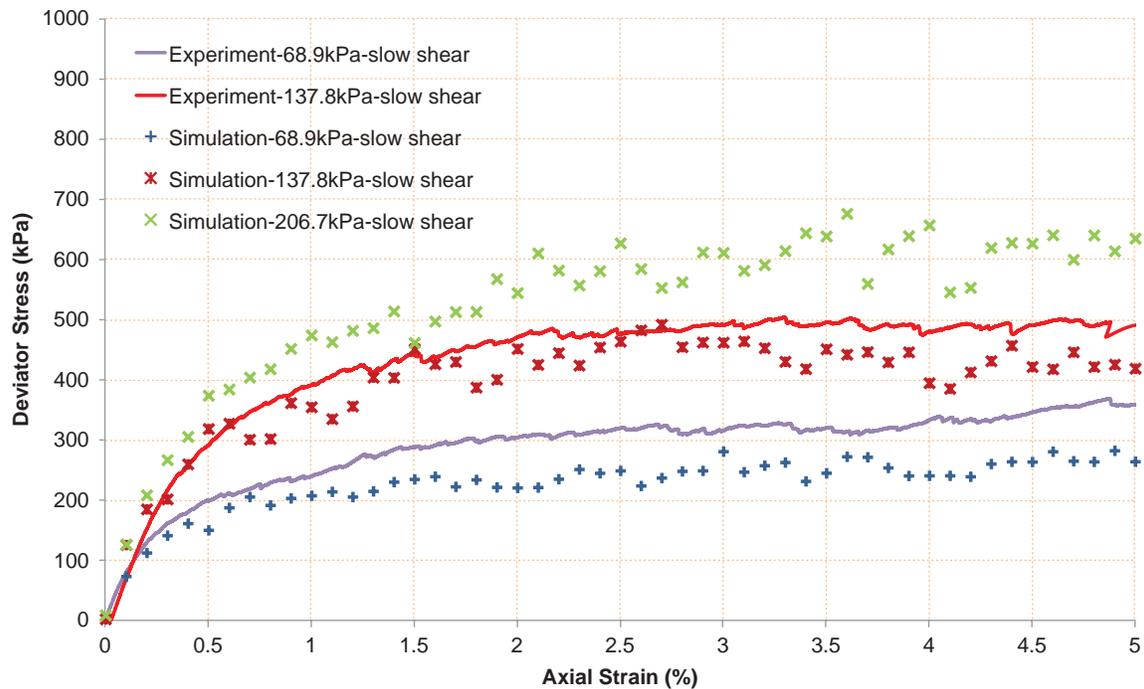


FIGURE 8 Comparisons of DEM simulations and strength results from slow loading triaxial tests.

experiments. Not only is the stress–strain response predicted by the DEM simulations, but also the trends in volumetric changes could be captured and quantified from the triaxial compression strength test simulations. The tested samples in the DEM simulations contracted first and then kept dilating because of the dense nature of the ballast samples after compaction. In agreement with typical triaxial compression behavior trends, the total volume decreased first because of the compression of the top platen but then increased later on because of particle rearrangements and shearing.

Figure 9 shows the changing geometry (i.e., bulging) of one of the ballast test samples captured from the side view through the acrylic chamber during triaxial testing and the deformation simulated by the DEM. Since the chain extensometer could not be used for on-specimen measurements during the lab test, a comparison of the volume change behavior between the simulation and the lab test cannot be made at this time. This behavior will be the subject of subsequent work dealing with repeated load permanent deformation testing of ballast.

DEM Simulations with Incremental Displacement Shearing Method

In the case of slow loading tests, each ballast sample was sheared at a rate of 0.102 mm/s, which took only 5 min to reach the targeted 5% of axial strain in the laboratory. For the DEM simulations, 2.70×10^{-6} s of time step Δt was taken, resulting in as many as 1.11×10^8 time steps to reproduce a triaxial test. Accordingly, it took around 15 days to complete a single DEM simulation; it is never practical to repeat several DEM simulations to get meaningful averaged values of the results. Therefore, an alternative approach was adopted, referred to here as the incremental displacement shearing method (IDSM), to mimic the quasi-static

loading scheme with greatly saved computational resources and run time. The IDSM was successfully employed by Lee et al. to simulate triaxial compression tests of sand, although the terminology was not explicitly introduced (14). In the study by Lee et al., the top platen was allowed to move incrementally to shear the sand specimen rather than moving continuously, and the stress and strains were then calculated after the test specimen reached equilibrium under the new boundary condition. This dynamic relaxation scheme could shorten the run time to simulate quasi-static problems. Similar approaches for DEM simulations of triaxial tests can also be found in the literature (18, 20).

In this study, the 5% total axial strain (30.5 mm) was reached after 500 incremental displacement stages. At each stage, the top platen moved by 6.096×10^{-4} mm and then the simulation was continued while the whole system was reequilibrated before the next incremental displacement was applied. The IDSM could significantly reduce the computer processing time to around 30 h to complete each triaxial test simulation, which was a significant saving in computational resources.

Figure 10 presents the predictions of the DEM simulations obtained with the IDSM and compares them with the test results from the experimental study. The DEM predictions obtained from the IDSM match the experimental results closely. Applying quasi-static loading schemes instead of quick loading in the DEM simulations could still capture the same deformation trends of the actual test sample loaded under rapid shear conditions in the laboratory. This result implies that the loading rate has an insignificant effect on the measured ballast strength and DEM can reproduce these inherent characteristics. Figure 11 presents the stress paths traced in the laboratory rapid shear tests and the corresponding DEM simulation predictions from the IDSM. The friction angles calculated on the basis of the peak deviator stresses are given in Table 2.

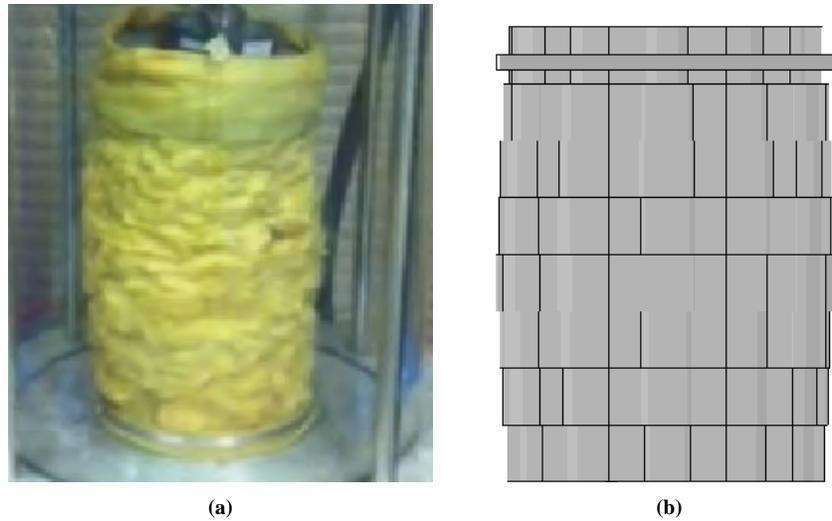


FIGURE 9 Deformed ballast sample configuration from (a) experiment and (b) DEM simulation at rapid shear test with 68.9-kPa confining pressure.

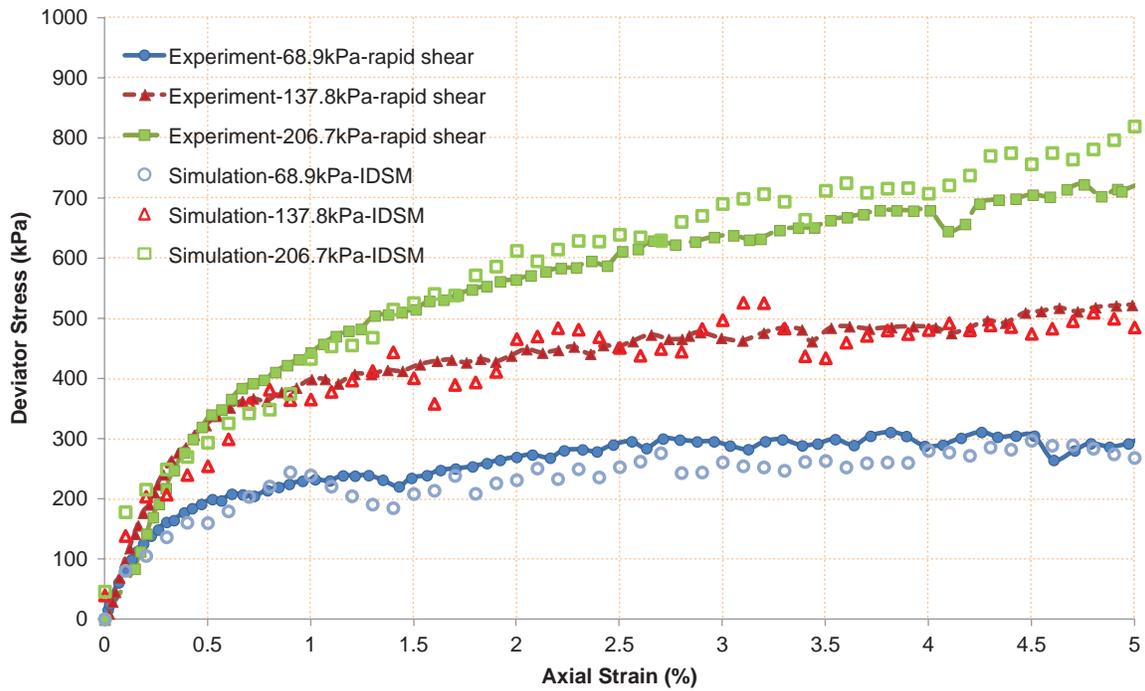


FIGURE 10 Comparisons of DEM predictions and triaxial rapid shear strength test results.

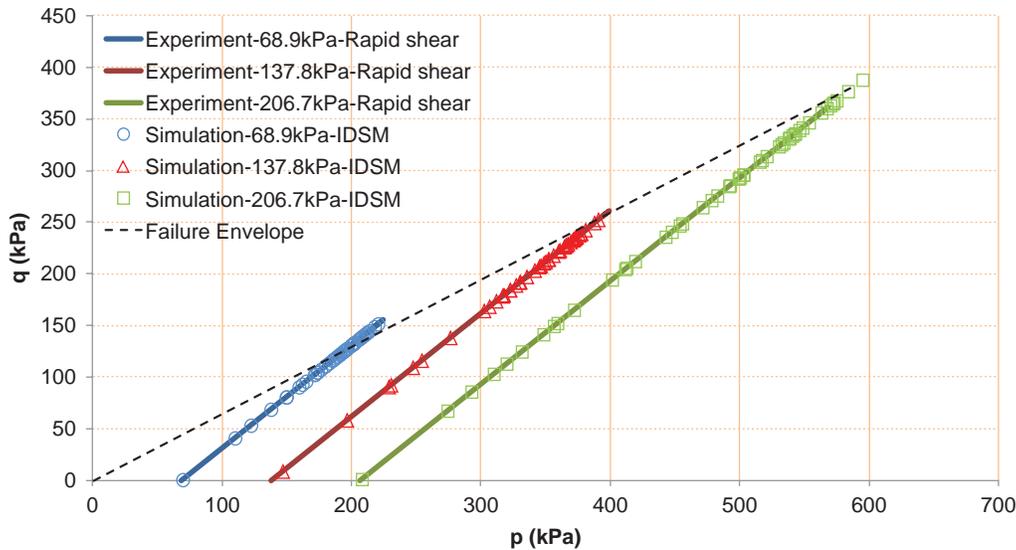


FIGURE 11 Stress paths of triaxial rapid shear strength tests and DEM (p = mean normal stress; q = axial deviator stress).

CONCLUSIONS

Both experimental and numerical simulation results of large-scale ballast triaxial strength tests conducted recently at the University of Illinois are presented. A realistic railroad ballast numerical model, developed on the basis of the polyhedral DEM, was used to simulate laboratory triaxial tests for compressive strength behavior of a limestone ballast material. Both slow and rapid shear loading rates were considered in the experimental program under a displacement control mode. The following conclusions can be drawn from this study:

1. The shear loading rate applied in triaxial compression tests was found to have an insignificant effect on the limestone ballast shear strength. Both traditional slow loading, 1% strain per minute, and a rapid shear test, 5% strain per 1 s, yielded similar stress–strain curves when ballast specimens were tested monotonically at three different confining pressures.
2. After the initial conditions of individual test samples were adequately addressed, the DEM simulations were conducted by two approaches: the continuous shearing method and the IDSM. The continuous shearing DEM scheme used the previously calibrated model parameters for predicting the shear strength test results. The

IDSM was a more robust approach that not only reproduced closely the shear strength test results at different confining pressures but also saved significant computational resources and shortened the DEM simulation run time to make it more practical and beneficial.

3. The DEM simulation platform developed at the University of Illinois could model confinement and applied stress conditions on cylindrical specimens and capture the stress–strain behavior from a realistic ballast DEM simulation. The DEM simulation platform, currently being further developed, has the potential to be a quantitative tool to predict ballast field performance.

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TABLE 2 Calculated Friction Angles from Laboratory Tests and DEM Simulations

Confining Pressure (kPa)	Laboratory Test (degrees)		DEM Simulation (degrees)	
	Rapid Shear ^a	Slow Shear ^b	IDSM ^a	Continuous ^b
68.9	44	45	43	43
137.8	41	40	41	40
206.7	40	—	40	39

NOTE: — = not included in testing.

^aRapid shear tests were simulated with the IDSM.

^bSlow shear tests were simulated by using continuous shearing (IDSM could also be used in this case).

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